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Dual Reciprocity Boundary Face Method for Nonhomogeneous Problems

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The boundary face method (BFM)[1] is coupled with the dual reciprocity method (DRM)[2] to solve nonhomogeneous potential problems and nonhomogeneous elasticity problems. In this combined method, here called the dual reciprocity boundary face method (DRBFM), the variable shaped radial basis function (RBF)[3] is applied to approximate the nonhomogeneous term to improve the stability of the approximation. Furthermore, several variation schemes for the shape parameter of the RBF have been proposed. In the analysis of large scale problems and problems on thin structures, the condition number of the interpolation matrix generated by the proposed RBF is much smaller than conventional cases.

In DRBFM, both boundary integration and variable approximation are performed in the parametric boundary faces. The integrand quantities are calculated directly from boundary faces rather than from the standard elements as in the dual reciprocity boundary element method (DRBEM), thus geometric errors can be avoided. In most of the popular computer aided design (CAD) packages, the solid models are of boundary representation (B-rep) and functions to obtain the geometric data of boundary faces are available. Thus, the DRBFM can be directly linked with CAD packages.

In most engineering problems, small features are usually omitted in the finite element analysis, as the degree of freedoms increases considerably if these small features are considered in the grid model. The geometrical simplification often results in poor accuracy of stress at places of small features. The local stresses are usually more concerned by design engineers. In the implementation of the DRBFM, analysis is directly performed on the exact geometric model rather than on the grid model which is generated to approximate the geometric model. Thus structures with feature of small size can be analyzed without any geometric simplification.

In the analysis of problems on thin shell structures, strain assumptions are usually adopted in most finite element method (FEM) analysis. Hundreds of element type had been already proposed based on different assumptions. These assumptions, however, may inevitably introduce additional errors and may limit the area of the application. Solid elements with proper aspect ratios should be used when high accuracy is demanded. This will, however, result in large FEM models, and it may be difficult to generate the FEM mesh for thin shell-like structures if the geometry is complicated. In contrast, the analysis by our method is performed directly on solid models without any difficulty. Nevertheless, the stability of the interpolation applied in the DRM is another problem in the DRBFM.

In the DRM, global supported RBF is the most widely adopted approximation function. This kind of RBF, however, suffers from an ill-conditioning matrix which made the interpolation unstable and thus the result becomes unreliable[4]. The shape parameter of the RBF is of great importance to the stability and accuracy of the interpolation. A flatter shaped RBF usually results in better interpolation accuracy. The ill-conditioning interpolation matrix, however, made the interpolation more unstable. In the other hand, less accurate but more stable result can usually be obtained by applying a sharper shaped RBF. To circumvent the ill-conditioning problem, the variable shaped RBF is applied in the DRBFM. In the variable shaped RBF, the shape parameter of the variable shaped RBF varies with the interpolation point. We have proposed several variation schemes for the variable shaped RBF to improve the stability of the interpolation without too much loss of the accuracy. With the variable shaped RBF, the nonhomogeneous elasticity problems even in extreme cases of thin shell structures can be directly analyzed.

The combination of the BFM and the DRM results in an efficient method for solving nonhomogeneous problems. A number of application examples, including those from real world product design, are presented. In these examples, the DRBFM is compared with the DRBEM and the FEM on both accuracy and stability.

References

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